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H.H. Billon

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H.H. Billon

Ship Structures and Materials Division
Aeronautical and Maritime Research Laboratory

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ABSTRACT

A method is described for the determination of the true shear rate in a concentric cylinder viscometer. A computer program based on the MacSporran technique is used. The program is tested on model as well as real fluids and is shown to be satisfactory. Shear rate determination in yield stress and time-dependent fluids is also discussed.

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Executive Summary

Materials with a complex rheology are often encountered during investigation of the flow behaviour of explosives. These materials may be studied with a concentric cylinder rotational viscometer. This type of viscometer allows ease of sample insertion and ease of cleaning after use.

However, there is a problem associated with determining the true shear rate in a concentric cylinder viscometer. This is because the shear rate usually has to be calculated using a rheological model that describes the fluid under investigation. This fluid model is often unknown. In this paper a computational method is described. This method permits determination of the true shear rate from experimental data without recourse to a fluid model.

The method consists of determining the shear rates from the measured values of the shear stress and angular velocity. The shear rate at each data point is determined from an integral equation using a technique outlined by MacSporran. A computer program based on this technique was written and tested on model fluids and real fluids and shown to be satisfactory. Shear rate determination in yield stress and time-dependent fluids is also discussed.

Contents

1. Introduction

In examining the flow behaviour of explosives it is often necessary to make an assessment of mixtures that are rheologically complex - An example is the suspension of one explosive in another, as occurs with suspensions of either RDX [1,2] or TATB [3] in molten TNT. At AMRL such studies are carried out in a Haake RV2 viscometer which operates on the concentric cylinder principle [4]. Such geometry allows ease of sample insertion and, after use, ease of cleaning. However, there is a problem associated with determining the true shear rate at the spinning rotor. This arises because the true shear rate usually has to be calculated using a rheological model that describes the fluid under investigation. This fluid model is often unknown.

In this paper a computational method is described. This method determines true shear rates from experimental data without recourse to a fluid model.

2. Description of the Problem

For any fluid the true shear rate $\dot{\gamma}$ cannot easily be calculated unless there is only a very small gap between the bob and the cup. In this case the shear rate approaches a value given by

$$\dot{\gamma} = R_1\Omega / R_2 - R_1 \qquad (1)$$

Where $\dot{\gamma}$ is the shear rate, R_1 is the bob radius, R_2 is the cup radius and Ω is the angular velocity of the bob. This relationship is independent of fluid properties.

Experimentally, however, it is impractical to use extremely small gaps, especially when suspensions of large particles are being investigated [4].

The problems associated with the determination of true shear rate are further exacerbated by the fact that many widely promoted, commercially available software packages sold by instrument manufacturers for automatic determination of shear rate erroneously employ the equation:

$$\dot{\gamma} = 2\Omega\varepsilon^2 / \left(\varepsilon^2 - 1\right) \qquad (2)$$

which is strictly only valid for the determination of shear rates in Newtonian fluids. Here ε is the ratio of cup to bob radii. This equation is a special case of a more general equation

$$\dot{\gamma} = 2\left(\Omega/n\right)\varepsilon^{2/n} / \left(\varepsilon^{2/n} - 1\right) \qquad (3)$$

which is valid for any power-law fluid. For a Newtonian fluid, $n = 1$.

Not all fluids can be described as "power-law fluids" whose behaviour follows equation (3). Some fluids exhibit a yield stress which can lead to incomplete shearing in the annular gap of the viscometer; a plot of measured rotational speed versus shear rate cannot always be described by a simple model and this is a further complication [6].

For some yield stress fluids the shear rate is given by a relationship such as

$$\dot{\gamma}/\dot{\gamma}_n = \frac{1-\left(R_1/R_2\right)^2}{1-\left[1/\left\{\left(R_y/R_1\right)^2-1\right\}\right]\ln\left(R_y/R_1\right)^2} \tag{4}$$

These are called "Bingham plastics" [5]. Here $\dot{\gamma}_n$ is the apparent shear rate and R_y is the yield radius.

Krieger and Elrod [7] express the shear rate in a concentric cylinder viscometer in the form of an Euler-Maclaurin series,

$$\dot{\gamma} = \left(\Omega/\ln\varepsilon\right)\left[1+\ln\varepsilon\left(d\ln\Omega/d\ln\tau\right)+\left(\left(\ln\varepsilon\right)^2/3\Omega\right)\left(d^2\Omega/d\left(\ln\tau\right)^2\right)-\ldots\right] \tag{5}$$

Such an expression does not require any assumptions to be made about the fluid model and may be used as the basis for a technique to determine the true shear rate in a concentric cylinder viscometer. This method would require the determination of derivatives in the torque versus rotation speed curve that is obtained as raw data. Techniques involving differentiation have been used by e.g. Nguyen et al. [6] and Krieger and Maron [9]. However, differentiation of discrete data can be noisy and inaccurate, particularly when high order derivatives must be determined. This problem could be overcome if a technique involving integration could be used.

3. Determination of the Shear Rate by Solution of an Integral Equation

3.1 Description of the Method

The approach followed is outlined by MacSporran in [8]. The shear rate is evaluated by solving the integral relationship

$$\Omega = 0.5\int_{\tau_1}^{\tau_2}\left(f(\tau)/\tau\right)d\tau \tag{6}$$

Here τ_1 and τ_2 are the shear stresses at the inner and outer cylinders, respectively.

Experimental data is obtained in the form (Ω_n, τ_n), where Ω_n is the angular velocity of the bob and τ_n is the shear stress at either the bob or the cup. In this case τ_n will be the stress at the bob. Therefore the problem is to determine the shear rates $f(\tau_n)$ at each of the points (Ω_n, τ_n) by solution of the equation

$$\Omega(\tau_n) = \int_{c_n}^{\tau_n} w(\tau) f(\tau) d\tau \qquad (7)$$

Here $c_n = \tau_n / \varepsilon^2$ represents the shear stress at the cup and $w(\tau) = 1/2\tau$ is a 'weighting function'.

The integral in equation (7) is regarded as the contribution from a number of strips in $(\tau, f(\tau))$ space. For example, suppose we are testing a fluid in a viscometer that possesses a cup to bob radius ratio of $\varepsilon = 1.2$ and that we obtain the following (Ω, τ) measurements:

$(1.4382 \times 10^{-2}, 1)$, $(5.7527 \times 10^{-2}, 2)$, $(0.1294, 3)$, $(0.2301, 4)$. (See Table 1).

The angular velocity at τ_n = 4 Pa would consist of contributions from the following strips:

strip 1: 2.78 Pa $\leq \tau \leq$ 3 Pa
strip 2: 3 Pa $\leq \tau \leq$ 4 Pa

Here 2.78 Pa is the stress at the cup when the stress at the bob is 4 Pa (i.e. $4\text{Pa}/(1.2)^2$).

In general, for a bob stress $\tau = \tau_n$, the strips would possess the integration limits c_n and τ_i, where the τ_i lie between c_n and τ_n.

Table 1: An explanation of the method of determining the number of strips for each data point.

Measured Ω (Rad/s)	Measured τ (at the bob) (Pa)	τ at the cup (Pa)	No. of data points with bob τ values lying between the cup and bob τ values of this data point.	No. of strips for this data point.
1.4382×10^{-2}	1	0.69	0	1
5.7527×10^{-2}	2	1.39	0	1
0.1294	3	2.08	0	1
0.2301	4	2.78	1*	2

* Meaning the data point for which the measured τ value is 3 Pa. Because 2.78 < 3 < 4.

The shear rate $f(\tau)$ is approximated in each strip by a cubic polynomial which interpolates to $f(\tau)$ at four successive data points. Modifications are necessary to the interpolation points at the extremities of the data set [8]. In any event, the strip contribution, which is of the form $\int_{\tau_1}^{\tau_2} w(\tau)f(\tau)d\tau$, is rewritten (after approximating the shear rate by a cubic polynomial $P_3(x)$) in the form:

$$\int_{x_p}^{x_q} w(x)P_3(x)dx = \sum_{l=j-2}^{l=j+1} W_l^j f_l \qquad (8)$$

where x is the shear stress, x_p and x_q are the strip integration limits and the f_l are the desired shear rate values. Details of the method are given by MacSporran [8]. A brief summary is given below.

After summing contributions for all strips (for a given data point), the following expression is obtained for Ω_n:

$$\Omega_n = \sum_{k=j}^{k=n} \sum_{l=k-2}^{l=k+1} W_l^k f_l = \sum_{l=j-2}^{l=n+1} W_{nl} f_l \qquad (9)$$

Here $W_{nl} = \sum_{k=j}^{k=n} W_l^k$ are the composite weights.

The integration weights for the individual strips are obtained from:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ (x_1)^2 & (x_2)^2 & (x_3)^2 & (x_4)^2 \\ (x_1)^3 & (x_2)^3 & (x_3)^3 & (x_4)^3 \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{bmatrix} \qquad (10)$$

Here $M_r = \int_{x_p}^{x_q} x^{r-1} w(x)dx$.

Once the integration weights for the individual strips have been found they are used to calculate the composite weights. The shear rates $f_n = f(\tau_n)$ for the data points 1 to N are calculated from:

$$\begin{bmatrix} W_{11} & W_{12} & \rightarrow & W_{1N} \\ W_{21} & W_{22} & \rightarrow & W_{2N} \\ \downarrow & \downarrow & & \downarrow \\ W_{N1} & W_{N2} & \rightarrow & W_{NN} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \downarrow \\ f_N \end{bmatrix} = \begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \downarrow \\ \Omega_N \end{bmatrix} \qquad (11)$$

3.2 The Computer Program

A BASIC computer program (called "SHEARE.RAT") has been written to perform the above calculations. The program requires input of the radius ratio for the concentric cylinder viscometer. The shear stress values at the viscometer bob and the rotation speed values are also input. The number of strips for each data point are determined by finding all the bob shear stress values that lie between the cup shear stress value and bob shear stress value for the data point. The program then determines the moments and weights.

In order to test the program, (Ω_n, τ_n) pairs were generated for a power law fluid with a constitutive equation of the form $\tau = 3\dot{\gamma}^{0.5}$. These (Ω_n, τ_n) values were then used as 'data' for the program. The results are presented in Table 2. In this case perfect agreement was achieved between the actual shear rates and the shear rates as calculated by "SHEARE.RAT". Another test was conducted using the solids GR-S latex data of Krieger and Maron [9] and these results are presented in Table 3. For comparison, the shear rates as determined by the Tanner and Williams method [10, 11] as well as by MacSporran [8] using the present method, are tabulated. Very good agreement is again obtained.

Table 2: A Comparison of the Actual Shear Rate with the Shear Rate as Calculated by "SHEARE.RAT" for a Power Law Fluid with Equation $\tau = 3\dot{\gamma}^{0.5}$.

τ (Pa)	Ω (Rad/s)	$\dot{\gamma}$ Actual (s^{-1})	$\dot{\gamma}$ Calculated (s^{-1})
1	1.4382×10^{-2}	0.1111	0.1111
2	5.7527×10^{-2}	0.4444	0.4444
3	0.1294	1.000	1.000
4	0.2301	1.778	1.778
5	0.3595	2.778	2.778
6	0.5177	4.000	4.000
7	0.7047	5.444	5.444
8	0.9204	7.111	7.111
9	1.1649	9.000	9.000
10	1.4381	11.11	11.11
11	1.7402	13.44	13.44
12	2.0710	16.00	16.00
13	2.4305	18.78	18.78
14	2.8188	21.78	21.78
15	3.2359	25.00	25.00

Table 3: Shear rates calculated by the Tanner and Williams method [10, 11], by MacSporran [8] and by "SHEARE.RAT" for the solids GR-S latex data of Krieger and Maron [9].

τ (Pa)	Ω (Rad/s)	$\dot{\gamma}$ Tanner & Williams (s^{-1})	$\dot{\gamma}$ MacSporran (s^{-1})	$\dot{\gamma}$ Present Work (s^{-1})
3.807	0.03670	0.7283	0.7211	0.7211
5.706	0.1310	2.611	2.528	2.528
9.536	0.6020	11.43	11.40	11.40
15.24	1.940	35.60	35.81	35.81
19.03	3.120	57.61	57.17	57.17
22.83	4.660	85.33	85.66	85.66
26.65	6.410	116.7	116.3	116.2
30.45	8.330	150.9	151.1	151.1
34.25	10.45	187.5	188.1	188.1
38.07	12.51	222.6	220.4	220.7
53.27	21.70	385.4	387.2	387.0
68.48	32.20	568.9	563.1	563.8

The program was run on an IBM-compatible PC (80486 processor and 640k base memory) and it was found that there were restrictions, due to memory, on the number of data points that could be processed. The maximum number of points that could be analysed at a time was seventeen. The program required 4 seconds to analyse seventeen points. Improvements in the software and computer memory would increase the efficiency of this program.

3.3 Yield Stress Fluids

The program can be used to determine the shear rates for many types of fluids once adequate rotation speed and torque data have been obtained. However, care must be exercised when dealing with plastic fluids i.e. those exhibiting a yield stress. In such cases the integration method cannot always be used if a fixed viscometer radius ratio is assumed.

If it is necessary to determine the shear rate in a non time-dependent yield stress fluid, the following procedure is recommended. The yield stress must first be accurately determined. A simple method for directly measuring the yield stress is described in [12]. The next step is to determine whether there is partial or complete shearing in the viscometer gap. Partial shear will occur when $\tau_1/\varepsilon^2 < \tau_y < \tau_1$ and shear will only occur between the viscometer bob and a cylindrical surface at a radial distance $R_p = R(\tau_1/\tau_y)^{0.5}$. This distance is clearly less than the width of the entire gap. For the case of partial shear the shear rate may be precisely determined by means of the following equation [6].

$$\dot{\gamma}_1 = 2\Omega\left(d\ln\tau_1 / d\ln\Omega\right)^{-1} \qquad (12)$$

The derivative may be determined e.g. by graphical differentiation of a double log plot of the original stress versus rotation speed data. For the completely sheared situation of a time-independent yield stress fluid, the shear rate may be determined by means of the described computer program.

The case of time-dependent fluids that also exhibit a yield stress is much more complex. An approximate procedure for shear rate determination in this case is presented in [6].

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Horace Billon

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